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THE TECHNIQUE OF COMBINING INCOMPLETE JUDGMENTS OF THE RELATIVE POSITIONS OF N FACTS MADE BY N JUDGES

INVESTIGATIONS in the mental and social sciences often need to use measurements by relative position or rank or "order of merit." In such cases the series of facts to be ranked may not in entirety be known to any one judge. One judge may be able to form an opinion concerning only facts 1, 2, 3, 4, and 5; another judge may be able to rate only facts 1, 2, 6, 7, 8, and 19; a third judge may be able to rate only facts 6, 7, and 9. The investigator has then the problem of assigning the most probable order of merit to facts 1 . . . *n,* on the basis of a combination of several partial orders; or of assigning a fairly probable order obtainable without undue expense of time.

The technique for handling this problem has not, to the author's knowledge, been seriously considered, though, of course, the principles involved would be obvious to anybody familiar with the general theory of measurement by relative position. Since the problem will be met very often in applied psychology, I present here a fairly satisfactory procedure used in estimating the general intelligence imputed to each of thirty-four college freshman by the combined judgment of eighteen individuals who knew from 14 to 29 of the thirty-four, and rated those whom they knew. We have, that is, as our data, the facts of Table I., the ratings being made in accordance with the following instructions:

"Place a cross at the right of the name in the case of the individuals whom you know, including yourself. Then consider these individuals and mark with a 100 the man who, in your opinion, has the best intellect. Mark with a 1 the man who, in your opinion, has the least intellectual ability. Mark with a 99 the next to the best for intellect, and with a 2 the next to the worst. Continue marking down from the top or up from the bottom until all the individuals that you know are marked. When two individuals seem to be closely alike in intellectual ability, give them each two marks (for example, if after marking the first two at the top 100 and 99, the next two seem to be closely alike in intellect, mark each of them '98–97'). Grade yourself without over-modesty, as impartially as you can."

For convenience the first step is to get a rough approximation to the true order by inspection, or by computing the median position for each individual, or otherwise. It is obvious, for example, from inspection that individuals 35, 20, 4, 40, and 2 are high-numbered and that individuals 1, 8, and 26 are low-numbered.

In the case in hand the rough approximation left individuals 19 and 33 out of account, since they were graded by only one individual, and could best be treated separately. The resulting order was that of the heading line of Table II.; and in accordance with it the facts of Table I. were rearranged to form Table II.

The next step is to get a nearer approximation by measuring each individual, beginning at the right in comparison with his neighbor to the left in Table II. This is done by finding the percentage of judges (using only those who rated the two individuals in question), who rated the former individual as lower numbered than the latter, and computing the amount of difference represented by this percentage. The latter computation in this case assumed that the variability of the opinions of the judges concerning the ability of an individual was approximately that of the so-called normal surface of frequency. Thus, 26 is ranked as lower-numbered than 34 by 6, as equal by 0, and as higher-numbered by 3. He is, by the assumption, .64 times the Median Deviation (P. E.) below 34, in the judgment of those who know the two. A table of differences corresponding to various divisions of opinion amongst 2, 3, 4, 5, 6, 7, 8, . . . *n* judges will be found useful. (Table III. is such a table up to 15 judges.) Table IV. gives the result of this series of computations[1]. Where the values are negative, the first rough approximate order evidently needs amendment.

It is amended by comparing the wrongly placed individuals with other individuals near whom they seem to belong until, so far as possible, a place can be found for them such that they are, in every case, so placed as to give + differences or zero differences. In connection with this amendment, it will be useful to compute also the differences from the two or three next neighbors in the new order. For, if an individual is rightly placed, he will not only be below the one placed above him in the order, but also below the two or three next above him. A process of trial and adjustment will be more economical than a rigid procedure in this work. The result in the case in hand is as shown in Table V.

[1] Cases of no division of opinion among the judges, which theoretically correspond to an infinite difference, should be assigned values that are reasonable in view of the number of judges and the character of the facts judged.

TABLE I

THE RELATIVE POSITIONS, IN GENERAL INTELLECT, ASSIGNED TO CERTAIN INDIVIDUALS OF A TOTAL GROUP OF 34 BY EACH OF 18 JUDGES Each Individual Judged is Represented by a Column. The Rankings of each Judge Are Represented by a Row.

	1	2	3	4	5	6	7	8	9	10	11	12	14	15	17	18	20	21	22	25	26	27	30	31	32	33	34	35	36	37	38	39	40	42
Judged by 2.	1	98	...	100	...	4	96	4	10	93	90	95	5	...	94	3	...	8	...	...	9	97	6	...	...	...	...	99	7	92	91	2	...	89
" 5.	8	...	97	99	90	96	...	79	...	93	...	83	91	88	98	89	...	...	92	...	80	95	86	87	85	...	...	94	84	...	...	81	100	...
" 6.	50	99	91	96	86	97	...	...	5	4	87	...	89	88	90	95	...	...	...	...	1	100	7	85	2	...	...	94	3	...	...	93	98	92
" 7.	1	100	...	98	...	84	97	82	83	92	91	95	85	81	96	90	...	86	93	87	80	94	...	...	...	...	89	99	3	88	...	2	...	...
" 10.	1	...	...	...	97	91	90	89	95	88	87	98	94	...	...	92	...	...	...	96	2	...	93	...	...	...	86	99	...	...	...	...	100	...
" 11.	14	99	93	100	88	91	92	1	86	90	89	97	87	13	12	11	...	5	6	3	2	...	95	8	94	...	9	98	4	...	...	7	96	10
" 12.	2	100	...	98	...	93	96	1	87	89	90	95	...	...	97	88	...	91	86	...	3	...	...	...	...	...	...	99	...	94	92	85	...	...
" 14.	3	96	95	94	92	90	97	7	6	9	93	...	8	12	99	92	...	...	89	4	...	...	...	...	...	...	...	98	5	...	...	1	100	...
" 18.	81	...	...	96	85	90	...	...	88	98	87	97	95	...	92	94	...	86	89	...	...	...	93	...	82	...	83	100	...	...	...	83	100	...
" 20.	1	96	...	99	...	...	...	...	...	8	4	...	5	6	94	2	99	93	...	...	...	95	...	3	...	...	...	100	7	...	...	92	97	...
" 21.	3	96	...	98	...	...	92	...	...	...	6	94	7	...	94	4	99	95	8	...	1	91	2	...	9	...	5	100	...	...	10	...	97	...
" 27.	1	96	...	99	7	90	92	3	...	...	90	89	88	2	97	4	98	95	94	9	12	...	91	10	6	...	11	100	6	...	8	93	5	...
" 30.	1	...	...	90	...	94	...	...	5	95	...	...	...	...	...	5	...	...	...	3	2	98	97	4	7	...	93	...	...	...	...	...	100	...
" 31.	1	96	88	99	87	95	86	2	9	12	3	13	94	4	91	92	98	7	11	8	6	97	93	90	99	...	5	99	10	...	93	3	100	...
" 32.	5	94	...	99	...	100	...	...	92	88	89	...	87	...	90	91	...	85	...	2	1	98	97	85	86	...	93	95	...	...	86	3	4	...
" 33.	1	85	97	96	...	94	3	5	90	7	8	...	93	...	...	89	98	86	...	87	99	84	6	95	92	84	4	100	...	11	88	12	91	2
" 35.	5	100	...	...	1	...	98	...	...	93	...	98	...	...	96	4	...	6	7	...	3	...	2	...	95	...	...	...	...	...	97	...	99	...
" 40.	2	94	...	3	6	...	...	...	...	8	...	...	7	4	89	...	99	92	95	...	5	94	98	...	...	...	...	100	1	...	91	96	97	...

TABLE II

THE FACTS OF TABLE I. REARRANGED IN AN APPROXIMATE ORDER BY INSPECTION. Each Individual Judged is Represented by a Column. The Ratings of each Judge Are Represented by a Row.

	35	4	2	40	7	20	3	6	17	12	30	27	10	21	31	32	38	5	18	11	33	39	22	14	42	37	9	25	15	36	1	8	34	26
Judged by 2.	99	100	98	...	96	...	...	4	94	95	6	97	93	8	...	...	91	...	3	90	...	2	...	5	89	92	10	...	...	7	1	4	...	9
“ 5.	94	99	...	100	...	...	97	96	98	83	86	95	93	...	87	85	...	90	89	...	...	81	92	91	...	...	...	...	88	84	8	79	...	80
“ 6.	94	96	99	98	...	...	91	97	90	...	7	100	4	...	85	2	...	86	95	87	...	93	...	89	92	...	5	...	88	3	50	...	...	1
“ 7.	99	98	100	...	97	...	...	84	96	95	...	94	92	86	...	...	...	...	90	91	...	2	93	85	...	88	83	87	81	3	1	82	89	80
“ 10.	99	...	...	100	90	...	...	91	...	98	93	...	88	...	...	...	...	97	92	87	...	...	...	94	...	...	95	96	...	...	1	89	86	2
“ 11.	98	100	99	96	92	...	93	91	12	97	95	...	90	5	8	94	...	88	11	89	...	7	6	87	10	...	86	3	13	4	14	1	9	2
“ 12.	99	98	100	...	96	...	...	93	97	95	...	...	89	91	...	...	92	...	88	90	...	85	86	...	...	94	87	...	...	...	2	1	...	3
“ 14.	98	94	96	100	97	...	95	90	99	...	...	...	9	...	...	...	...	92	92	93	...	1	89	8	...	...	6	4	12	5	3	7	...	...
“ 18.	100	96	...	100	...	...	...	90	92	97	93	...	98	86	...	82	...	85	94	87	...	84	89	95	...	...	88	...	...	91	81	...	84	...
“ 20.	100	99	96	97	...	99	...	...	94	...	...	95	8	93	3	...	...	...	2	4	...	92	...	5	...	...	...	...	6	7	1	...	...	...
“ 21.	100	98	96	97	92	99	...	...	94	94	2	91	...	95	...	9	10	...	4	6	...	...	8	7	...	...	...	...	...	...	3	...	5	1
“ 27.	100	99	96	5	92	98	...	90	97	89	91	...	...	95	10	6	8	7	4	90	...	93	94	88	...	...	...	9	2	6	1	3	11	12
“ 30.	...	90	...	100	...	...	...	94	...	...	97	98	95	...	4	7	...	...	5	...	...	...	...	...	...	...	5	3	...	...	1	...	93	2
“ 31.	99	99	96	100	86	98	88	95	91	13	93	97	12	7	90	89	93	87	92	3	...	3	11	94	...	...	9	8	4	10	1	2	5	6
“ 32.	95	99	94	4	...	...	...	100	90	...	97	98	88	85	85	96	86	...	91	89	...	3	...	87	...	...	92	2	...	...	5	...	93	1
“ 33.	100	96	85	91	3	98	97	94	...	...	6	84	7	86	95	92	88	...	89	8	84	12	...	93	2	11	90	87	...	...	1	5	4	99
“ 35	...	...	100	99	98	...	...	...	96	98	2	...	93	6	...	95	97	1	4	...	...	...	7	...	...	...	...	...	...	...	5	...	...	3
“ 40.	100	3	94	97	...	99	...	...	89	...	98	94	8	92	...	...	91	6	...	...	...	96	95	7	...	...	...	...	4	1	2	...	...	5

TABLE III

THE DIFFERENCES IN TERMS OF THE MEDIAN DEVIATION CORRESPONDING TO ANY GIVEN DIVISION OF OPINIONS AMONG 2, 3, 4, 5, 6, . . ., 15 JUDGES

N = No. of judges; D = Nature of the division among the judges; Δ = Corresponding difference in terms of the Median Deviation (P. E.).

N	D	Δ	N	D	Δ	N	D	Δ
2	2–0	∞	8	8–0	∞	12	12–0	∞
2	1–1	0	8	7–1	1.71	12	11–1	2.05
			8	6–2	1.00	12	10–2	1.43
3	3–0	∞	8	5–3	.47	12	9–3	1.00
3	2–1	.64	8	4–4	0	12	8–4	.64
						12	7–5	.31
4	4–0	∞	9	9–0	∞	12	6–6	0
4	3–1	1.00	9	8–1	1.81			
4	2–2	0	9	7–2	1.13	13	13–0	∞
			9	6–3	.64	13	12–1	2.11
5	5–0	∞	9	5–4	.21	13	11–2	1.51
5	4–1	1.25				13	10–3	1.10
5	3–2	.38	10	10–0	∞	13	9–4	.74
			10	9–1	1.90	13	8–5	.43
6	6–0	∞	10	8–2	1.25	13	7–6	.14
6	5–1	1.43	10	7–3	.78			
6	4–2	.64	10	6–4	.38	14	14–0	∞
6	3–3	0	10	5–5	0		13–1	2.17
							12–2	1.58
7	7–0	∞	11	11–0	∞		11–3	1.17
7	6–1	1.59	11	10–1	1.99		10–4	.84
7	5–2	.84	11	9–2	1.35		9–5	.54
7	4–3	.26	11	8–3	.89		8–6	.26
			11	7–4	.52		7–7	0
			11	6–5	.17			

We may now proceed to correct this amended order by assigning each individual an amount of difference from individual No. 1, using the comparison of him with each of his four neighbors (in the direction of No. 1) in Table V. Thus No. 8 is 1.25 from 1, No. 26 is .64 from 1 by direct comparison, and 2.38 via No. 8, and averages 1.51. No. 36 is .89 from 1 by direct comparison, and 2.09 via 8, and 1.51 via 26, and averages 1.50. No. 15 is theoretically an infinite distance from No. 1 by direct comparison. Call this, in view of the small number of the judges concerned, 2.00. No. 15 is 1.89 from 1 via 8; 1.77 via 26; 2.14 via 36. It therefore is on the average 1.95. No. 25 is 2.84 from 1 via 8; 2.51 via 26; 1.12 (1.50–.38) via 36; and 2.33 via 15. It therefore averages 2.20. The resulting distances of each individual from No. 1 are as given in Table VI.

The use of three indirect measures, rather than one or two or four or five, is arbitrary. The estimate could be made more reliable by weighting each comparison according to the square-root of the number of judges represented by it. The resulting order should not without some consideration be used instead of the order derived from comparison with next neighbors only; for the next-neighbor comparison, though not utilizing the data fully, is simple and unambig-

TABLE IV

THE DIFFERENCES BETWEEN SUCCESSIVE INDIVIDUALS IN TABLE II., ON THE BASIS OF DIRECT COMPARISONS.
1.00 = THE MEDIAN DEVIATIONS OF THE OPINIONS OF N JUDGES CONCERNING ANY GIVEN INDIVIDUAL

	34	8	1	36	15	25	9	37	42	14	22	39	33	11	18	5	38	32	31	21	10	27	30	12	17	6	3	20	7	40	2	4	35
26	.64																																
34		−.64																															
8			−1.25																														
1				.89																													
36					.64																												
15						.38																											
25							1.00																										
9								1.00																									
37									−∞																								
42										.00																							
14											−.64																						
22												−1.13																					
39														.43																			
33	(The data are inadequate for measuring the differences of No. 33.)																																
11															−.26																		
18																00																	
5																	.64																
38																		64															
32																			.47														
31																				−.32													
21																					.52												
10																						∞											
27																							−1.81										
30																								.64									
12																									19								
17																										−.52							
6																											.64						
3																												.64					
20																													−∞				
7																														1.71			
40																															−.17		
2																																.43	
4																																	.40
35																																	

THE DIFFERENCE BETWEEN NEIGHBORING INDIVIDUALS, ON THE BASIS OF DIRECT COMPARISON. 1.00=THE MEDIAN DEVIATION

		8	26	36	15	25	34	9	42	14	37	39	32	31	18	5	11	21	22	10	38	30	6	12	7	3	17	27	2	40	4	20	35
34	1	1.25	.64	.89	∞	1.13	1.90	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
33	8	....	1.13	.84	.64	1.59	.64	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
32	26	....	....	.00	.26	1.00	.64	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
31	36	....	....	....	.64	−.38	.38	.84	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
30	15	....	....	....	....	.38	1.00	.38	.00	1.13	∞	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
29	25	....	....	....	....	....	.47	1.00	.00	1.00	.00	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
28	34	....	....	....	....	....	....	.47	.00	1.13	.00	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
27	9	....	....	....	....	....	....	....	.00	.78	1.00	−1.81	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
26	42	....	....	....	....	....	....	....	....	.00	∞	.00	.64	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
25	14	....	....	....	....	....	....	....	....	....	.64	.74	−.64	−∞	....	....	−.14	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
24	37	....	....	....	....	....	....	....	....	....	....	1.00	?	?	.00	?	1.00	....	.00	....	....	....	....	....	....	....	....	....	....	....	....	....	....
23	39	....	....	....	....	....	....	....	....	....	....	....	.47	.47	1.51	.47	.43	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
22	32	....	....	....	....	....	....	....	....	....	....	....	....	.47	−.17	.26	−.47	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
21	31	....	....	....	....	....	....	....	....	....	....	....	....	....	.64	.38	.84	−.32	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....
20	18	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.00	.26	.00	.00	....	....	....	....	....	....	....	....	....	....	....	....	....	....
																	.84																
19	5	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.64	.47	.21	....	....	....	....	....	....	....	....	....	....	....	....	....
17	11	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.17	.47	.31	.84	....	....	....	....	....	....	....	....	....	....	....	....
16	21	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.64	.52	.64	.00	....	....	....	....	....	....	....	....	....	....	....
15	22	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.64	.64	.00	1.00	....	....	....	....	....	....	....	....	....	....
14	10	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.47	.31	1.00	1.13	....	....	....	....	....	....	....	....	....
13	38	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.23	1.43	1.43	.84	....	....	1.81	....	....	....	....	....
12	30	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.00	.64	.00	.38	....	....	....	....	....	....	....
11	6	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.64	.64	.64	.52	.47	....	....	....	....	....
10	12	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.42	.64	.19	.38	....	....	....	....	....
9	7	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	1.00	.21	.38	1.90	1.71	....	∞	....
8	3	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.38	.00	1.25	1.43	.64	∞	....
7	17	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.64	1.43	1.43	1.59	∞	....
6	27	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.43	1.13	.89	∞	1.00
5	2	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.17	.43	∞	.64
4	40	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	−.14	1.43	.15
3	4	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	.32	.40
2	20	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	∞
1	35	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....	....

uous in its meaning. Perhaps some simple system of overweighting the next-neighbor comparison would be advisable. On the whole, however, we have in this technique an economical and fairly reliable means of combining partial orderings.

When the number of judges represented in a comparison is large there is less need of making the final correction by indirect comparisons. When the number of judges is small it may be well to compare *viâ* five or six near neighbors.

TABLE VI

DIFFERENCES OF EACH INDIVIDUAL FROM THE LEAST INTELLECTUAL, NO. 1, IN TERMS OF THE MEDIAN DEVIATION OF THE OPINIONS

Individual	Distance Above No. 1	Individual	Distance Above No. 1
1	0	21	4.02
8	1.25	22	4.30
36	1.50	10	4.44
26	1.51	30	4.54
15	1.95	38	4.84
25	2.20	6	5.39
42	2.31	7	5.61
34	2.41	12	5.76
9	2.69	17	6.08
39	2.90	3	6.24
32	2.94	27	6.27
14	3.13	2	7.30
37	3.55	4	7.48
5	3.76	40	7.51
18	3.84	35	8.52
31	3.98	20	8.58
11	4.00		

E. L. THORNDIKE

TEACHERS COLLEGE,
COLUMBIA UNIVERSITY.

ETHICS AS SCIENCE AND AS ART

I

ALMOST any undertaking may be and has been described as a science, just as almost any one may and does call himself a professor.

When I propose to consider ethics as a science it is in the narrowed sense of "ordered knowledge of natural phenomena and of the relations between them." I have in mind also (*a*) the saying of Svante Arrhenius that "science implies measurement," (*b*) the practise of experimental research instituted by Francis Bacon, and (*c*) Galileo's